Recurrence relations for all kinds of the Stirling numbers, "Stx" { x ∈ ℕ : x > 0 }

St1(r; n+1, k) = St1(r; n, k-1) + St1(r; n, k) [$1(n) + 1(r-1)$]

St2(r; n+1, k) = St2(r; n, k-1) + St2(r; n, k) [$1(k) + 1(r-1)$]

St3(r; n+1, k) = St3(r; n, k-1) + St3(r; n, k) [$1(n+k) + 2(r-1)$]

St4(r; n+1, k) = St4(r; n, k-1) + St4(r; n, k) [$2(n+k) + 4(r-1)$]

St5(r; n+1, k) = St5(r; n, k-1) + St5(r; n, k) [$4(n+k) + 8(r-1)$]

St6(r; n+1, k) = St6(r; n, k-1) + St6(r; n, k) [$8(n+k) + 16(r-1)$]

General formula for Stirling and r-Stirling numbers "x", greater than or Equal to "3" : x ≥ 3

Example 1:

Determining the value of "St5(r; n+1, k) = St5(1; 5+1, 3)" in the table of Stirling numbers of the fifth kind.

Stirling numbers` kind, Stx = 5 : Restricted value, r = 1 : n = 5 : k = 3

St5(r; n+1, k) = St5(r; n, k-1) + St5(r; n, k) [$4(n+k) + 8(r-1)$]

St5(1; 5+1,3) = St5(1; 5, 3-1) + St5(1; 5, 3)[$4(5+3) + 8(1-1)$]

76800 = 15360 + 1920 [$4(5+3) + 8(r-1)$]

76800= 15360+ 1920[$4(5+3) + 8\cdot(1-1)$]

r = 1

Table of St5

n\k	1	2	3	4	5	6
1	1					
2	8	1				
3	96	24	1			
4	1536	576	48	1		
5	30720	15360	1920	80	1	
6	737280	460800	76800	4800	120	1

Example 2:

Determining the value of "St5(r; n+1, k) = St5(3; 5+1, 3)" in the table of Stirling numbers of the fifth kind.

Stirling numbers` kind, Stx = 5 : Restricted value, r = 3 : n = 5 : k = 3

St5(r; n+1, k) = St5(r; n, k-1) + St5(r; n, k) [$4(n + k) + 8(r - 1)$]

St5(3; 5+1,3) = St5(3; 5, 3-1) + St5(3; 5, 3)[$4(5 + 3) + 8(3 - 1)$]

460800= 129024+ 6912[$4(5 + 3) + 8\cdot(3 - 1)$]

n\k	1	2	3	4	5	6	
							r = 3
1	1						
2	24	1					
3	672	56	1				
4	21504	2688	96	1			
5	774144	129024	6912	144	1		
6	30965760	6451200	460800	14400	200	1	

Table of r-St5

Example 3:

Determining the value of "St3(r; n+1, k) = St5(1; 4+1, 3)" in the table of Stirling numbers of the third kind or **Lah** numbers .

Stirling numbers` kind, Stx = 3 : Restricted value, r = 1 : n = 4 : k = 3 : Lah number

St3(r; n+1, k) = St3(r; n, k-1) + St3(r; n, k) [$1(n + k) + 2(r - 1)$]

St3(1; 4+1,3) = St3(1; 4, 3-1) + St3(1; 4, 3)[$1(4 + 3) + 2(1 - 1)$]

120 = 36 + 12 [$1(4 + 3) + 2(1 - 1)$]

120= 36+ 12·[$1(4 + 3) + 2\cdot(1 - 1)$]

n\k	1	2	3	4	5	6
1	1					
2	2	1				
3	6	6	1			
4	24	36	12	1		
5	120	240	120	20	1	
6	720	1800	1200	300	30	1

r = 1

Table of St3 = lah triangle

And table of r-lah nambers in case r = 2

n\k	1	2	3	4	5	6
1	1					
2	4	1				
3	20	10	1			
4	120	90	18	1		
5	840	840	252	28	1	
6	6720	8400	3360	560	40	1

r = 2

Table of 2-St3 = 2-lah triangle

For example the value of n+1 = 5 : k = 3 : r = 2

$$90+18[1(4+3)+2\cdot(2-1)]=252$$

Key words: General formula for Stirling and r-Stirling numbers (x) ; Stirling and r-Stirling numbers**;** Stirling numbers of the fifth kind;

Mohammad Reza. Serajian Asl